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Improved tolerance allocation of optical systems based on compensator optimization

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Abstract
Tolerance allocation is very important in optical design because it has the significant effects on the as-built performance of optical system. In this paper, we propose an improved tolerance allocation method that takes into account all the structural parameters of the optical system as well as the compensator. Sensitivity coefficients of all the structural parameters are calculated based on the compensator optimization, and tolerances are assigned by solving the constrained optimization problem for minimizing the manufacturing cost while satisfying the image quality. Finally, the as built performance of the Cooke triplet is predicted by Monte-Carlo simulation and the feasibility of this method is demonstrated.

Keywords: Optical design, tolerance allocation, economic tolerancing, cost-based tolerancing, Monte-Carlo analysis, as-bulit performance

1. Introduction

The traditional optical design has two stages: optical system optimization and tolerancing. The goal of the first stage is to find the optical system with the best nominal performance, while not considering image quality degradation due to the manufacture and assembly errors [1]. However, the as-built performance of the optical system will be deteriorated due to manufacturing tolerance. In the second stage, a designer determines the tolerances of the input parameters including radius, thickness, surface tilt and decenter such that merit function of the optical system is within the acceptable range.

Tolerance analysis is the estimation of image quality degradation according to the pertubation of the input parameters. Tolerance allocation is the process of assigning tolerance for every structural parameter such that merit function is under specification. In tolerance allocation process, an iterative trial and error approach is generally used in which the tolerances are adjusted and the change of merit function is evaluated. Here static algorithm, for example Monte Carlo method, is usually used.

Compensation is the important concept in tolerancing. By adjusting some of the structural parameters such as a radius of curvature, thicknesses and decentering or tilting of a component, we can improve the degraded image quality of the system. This process is called compensation and the adjustable structural parameters are called compensator [2, 3].

Another important concept in tolerancing is "cost-based tolerancing" [4, 5]. In [6] introduced distortion as an index to evaluate the performance of the optical system and implement tolerance allocation for curvature, thickness, eccentricity and tilt and decenter for Cooke triplet.

In [7-8], the relationship between the structural parameters of an optical system and its corresponding emitting rays or optical path is described using a gradient matrix. This matrix can be used to evaluate the sensitivity of the optical system by performing a single ray trace.

In [9], an automated procedure for tolerance assignment is described to minimize manufacturing cost.

In [10], a simple and fast method for achieving a reasonable tradeoff between performance and cost is proposed. An example illustrates that only two or three Monte Carlo analyses are enough to solve the tolerance allocation problem.

In [11], a design of a zoom lens was applied to the tolerance allocation to meet multiple criteria.

In [12], "used-defined tolerancing" method, which ignores cross effect of the tolerance is proposed.

In [13], a methodology using the worst-case approach is proposed for tolerance analysis and tolerance allocation of optical systems.

There are few researches to consider the compensator in tolerance allocation. If the proper compensation is not considered in tolerance allocation, the tolerances might be tighter than necessary, and manufacturing cost can be increased [14]. Hence, we propose a method to allocate tolerances of optical system taking into account compensator.

In Section 2 an overview of the proposed tolerancing procedure is given. In Section 3, we take an example of Cooke triplet from [6] and implement our method in detail. In Section 4, yield rate is predicted and compared using Monte Carlo method. In Section 5, concluding remarks are given.

2. Theory Overview

We use the functional relationship between cost and tolerance proposed in [13],

$$\$ = \sum_{i=1}^{m} c_i = \sum_{i=1}^{m}\left[u_i + \frac{v_i}{(\Delta x_i)^2}\right] \quad (1)$$

where *\$* is the total cost of producing all the elements of the system and c_i is the cost of attaining the tolerance Δx_i. In addition, u_i is a constant coefficient that has no effect on the solution of Δx_is and reflects the cost related to tool, material, prior operations, and so on, while v_i determines the cost of producing an element at a specified tolerance level.

The relationship between the merit function Φ of the optical system and the structural parameters $x_1, x_2, \cdots x_n$ can be expressed as $\Phi=\Phi(x_1, x_2, \cdots x_n)$. Using Taylor's formula, and omitting terms of third and high order, we can write the equation as follows:

$$\Delta\Phi = \sum_{i=1}^{m}\frac{\partial\Phi}{\partial x_i}\Delta x_i + \frac{1}{2}\sum_{i,j=1}^{m}\frac{\partial^2\Phi}{\partial x_i \partial x_j}\Delta x_i \Delta x_j \quad (2)$$

where, Φ is a given merit function and

$\frac{\partial\Phi}{\partial x_i}, i = 1,2\cdots n$ is the first-order sensitivity coefficient,

$\frac{\partial^2\Phi}{\partial x_i \partial x_j}, i,j = 1,2\cdots n,$ second-order sensitivity coefficients.

If $i=j$, they are called as second-order sensitivity coefficients and if $i\neq j$, they are called as second-order cross sensitiviy coefficients.

Finite difference method is used to determine the sensitivity coefficients as follows:

$$\frac{\partial\Phi}{\partial x_i} \cong \frac{\Phi(x_i+\Delta x_i)-\Phi(x_i-\Delta x_i)}{2\Delta x_i}$$

$$\frac{\partial^2\Phi}{\partial x_i^2} \cong \frac{\Phi(x_i+\Delta x_i)+\Phi(x_i-\Delta x_i)-2\Phi(x_i)}{\Delta x_i^{\,2}} \quad (3)$$

$$\frac{\partial^2\Phi}{\partial x_i \partial x_j} \cong \frac{\Phi(x_i+\Delta x_i, x_j+\Delta x_j)-\Phi(x_i+\Delta x_i, x_j-\Delta x_j)}{4\Delta x_i \Delta x_j} - \frac{\Phi(x_i-\Delta x_i, x_j+\Delta x_j)-\Phi(x_i-\Delta x_i, x_j-\Delta x_j)}{4\Delta x_i \Delta x_j}$$

Our goal is to establish a methodology to find Δx_is such that the worst merit function value of an optical system ($\Delta\Phi_{worst}$) is less than an acceptable limit $\Delta\Phi_{spec}$.

$$\Delta\Phi_{worst} \leq \Delta\Phi_{spec} \quad (4)$$

Thus, the problem of tolerance allocation is to minimize the total cost *\$* while satistying the inequality constraints and boundary conditions for tolerances.

The tolerancing procedure is shown in Fig 1. First, the user inputs the merit function requirement and the boundary condtion for each tolerance. Second, sensitivity coefficients considering the compensator are calculated using Zemax's tolerancing function. The user carries out the sensitivity analysis for nominal system, gets the $\Phi(x_i+\Delta x_i)$, $\Phi(x_i-\Delta x_i)$ for all the structural parameters and determines the first order and second order sensitivity coefficients using equation (3). And by pertubating individual structural parameter one by one and analyzing tolerance sensitivity of the optical system and the second order cross sensitivity coefficients are determined. Finally, by solving the constrained optimization problem, the tollerances are allocated.

3. Example

We take an example of a Cooke triplet given in [6]. The focal length is 60 mm, F/5, and the field of view is $2\omega=24°$. The operating band is between 486 and 656 nm, and the dominant wavelength is 587 nm. The lens data are listed in Table 1, the layout and MTF are given in Fig 2 and Fig 3.

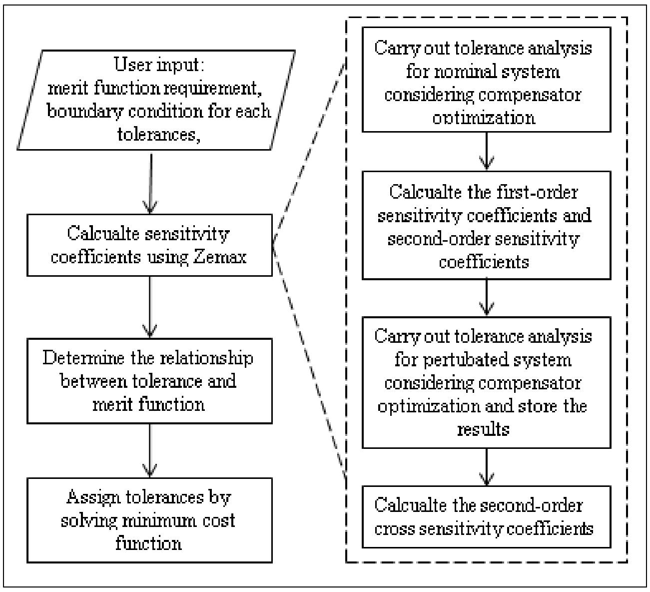


Fig 1: Tolerance allocation procedure

Table 1: Lens data of the example system

Surface	Radius (mm)	Thickness (mm)	Glass	Diameter (mm)
1	17.243	3.2	H-ZLAF89L	16
2	92.715	2.5		16
3	-545.8	2.0	H-F71	12
4	13.35	17.7	-	12
5	47.5	4.7	H-ZPK1A	24
6	-61.931	43.3	-	24
7	Infinity	-	-	-

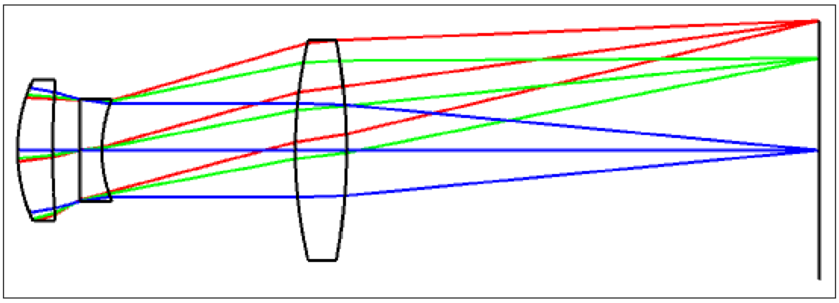


Fig 2: 2D layout of the example optical system

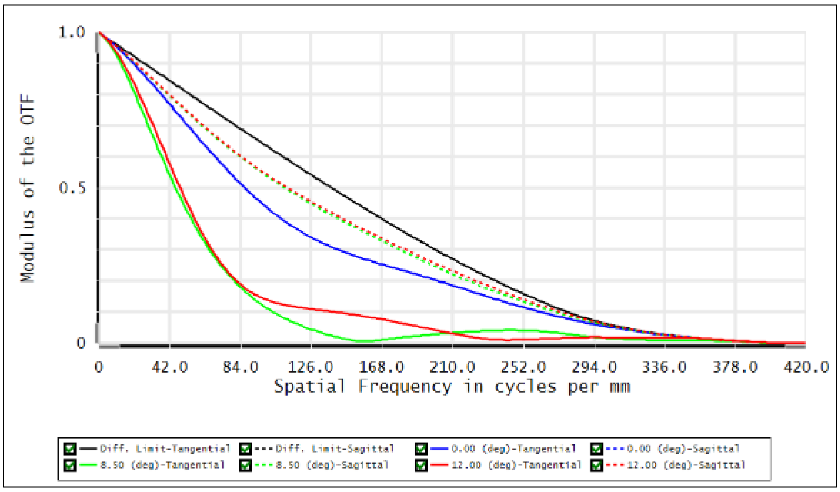


Fig 3: FFT MTF of the example optical system

We take both of u and v from equaton (1) as 1, and the RMS wavefront error as the merit function. The limit for radius in fringe is 1~5, thickness in millimeter is 0.01~0.1, and tilt in degree is 0.01~0.1, and decenter in millimeter is 0.01~0.2. We take the last thickness as the compensator and used damped least squares methods for compensator optimization.

Sensitivity coefficients calculated by tolerance analysis in Zemax considering compesator optimization are shown in Table 2~4. From the analysis result, the second-order cross sensitivity coefficients of the tilt and decenter are all zero, so table 4 only shows the coeffcents for curvature and thickness.

From table 2, all of the first-order sensitivity coefficients of the tilt and decenter are zero and the first-order sensitivity coefficients of radius and thickness are very small. From table 3, the second-order sensitivity coefficients of the tilt and decenter are relatively large and the first-order sensitivity coefficients of radius are very small. The second -order sensitivity coefficients of thickness are much larger than the first-order sensitivity coefficients. From table 4, cross sensitivity coefficients are different according to the derivative order. This is a different result from not considering the compensator.

Table 2: First Order Sensitivity Coefficient of the example optical system

Surface	Radius	Decenter	Tilt	Thickness
1	-6.83E-04	0	0	-9.52E-02
2	-5.86E-06	0	0	-
3	-3.18E-04	0	0	-2.06E-01
4	3.31E-04	0	0	-
5	3.20E-04	0	0	-1.20E-02
6	4.98E-05	0	0	-

Table 3: Second Order Sensitivity Coefficient of the example optical system

Surface	Radius	Decenter	Tilt	Thickness
1	6E-05	423.26	491.5	3.652
2	1.13E-05	11.1316	373.52	
3	3E-05	0.31354	648.32	4.16
4	6E-05	446.72	552.78	-
5	0	2.4492	9.5924	0.015
6	-1.2E-05	1.33622	8.896	-

Table 4: Second Order Cross Sensitivity Coefficient of the example optical system

Surface	r_1	r_2	r_3	r_4	r_5	r_6	t_1	t_2	t_3
r_1	-0.02614	0.000258	-7.5E-06	0.0198175	0.000745	1.75E-05	0.015865	0.014303	0.000148
r_2	0.007429	-0.00034	1.013E-05	-0.000288	-0.0008392	0.00023253	-0.00823	-0.00507	0.00047
r_3	-0.00813	0.000387	-1.25E-05	-0.001417	0.00121	-0.0002375	0.01008	0.006807	-0.00048
r_4	0.02112	-7.5E-06	0	-0.022668	0.000435	-0.000245	-0.00862	-0.0089	-0.00047
r_5	0.002518	-9.8E-05	5E-06	0.0013775	-0.0006525	0.00011	-0.00348	-0.00293	0.000117
r_6	9.8E-05	4.32E-05	-3.25E-06	-0.00133	0.00018775	-8.3E-05	0.000738	0.000239	-0.00012
t_1	2.4175	-0.11625	0.002	0.0305	-0.28325	0.10175	-2.7875	-1.624	0.18875
t_2	-5.125	0.09	0	2.35	0.405	-0.05	4.0925	3.52	-0.0425
t_3	-0.0505	-0.0055	0	0.15125	-0.01175	0.00725	-0.04925	0.017	0.015

Substituting these sensitivity coefficients into Eq. (2), we can determine the relationship between RMS wavefront error and the tolerances of the structural parameters. We taking specified limit of the RMS wavefront error as the $\lambda/10$ and solve the nonlinear optimization problem using genetic algoritm. Final results of tolerance allocation is shown in table 5.

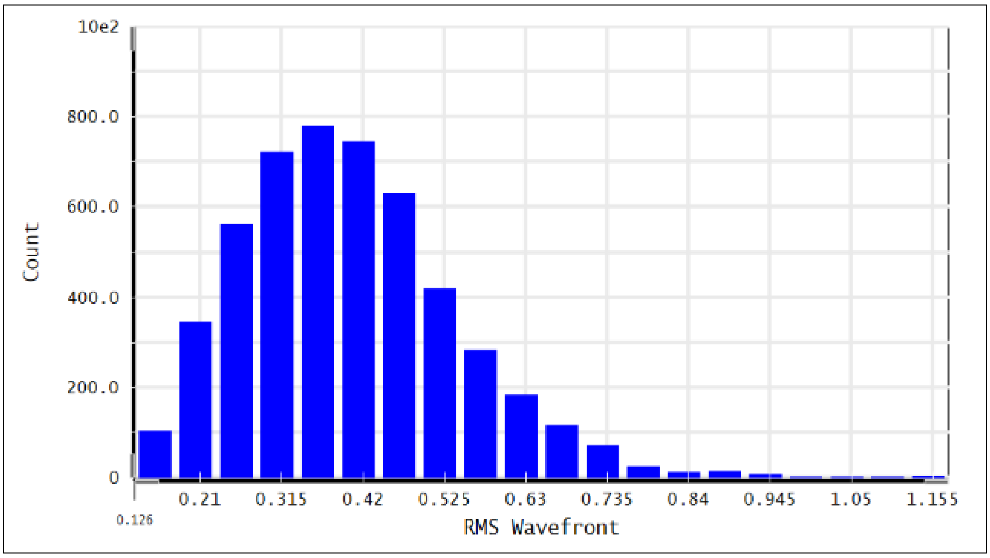
Table 5: Tollerace Allocation Result

Surface	Radius (fringe)	Decenter	Tilt	Thickness
1	5	0.0349	0.0336	0.0368
2	1	0.0867	0.0360	
3	5	0.1	0.0314	0.1
4	1	0.0344	0.0326	-
5	1	0.1	0.09	0.1
6	5	0.1	0.0917	-

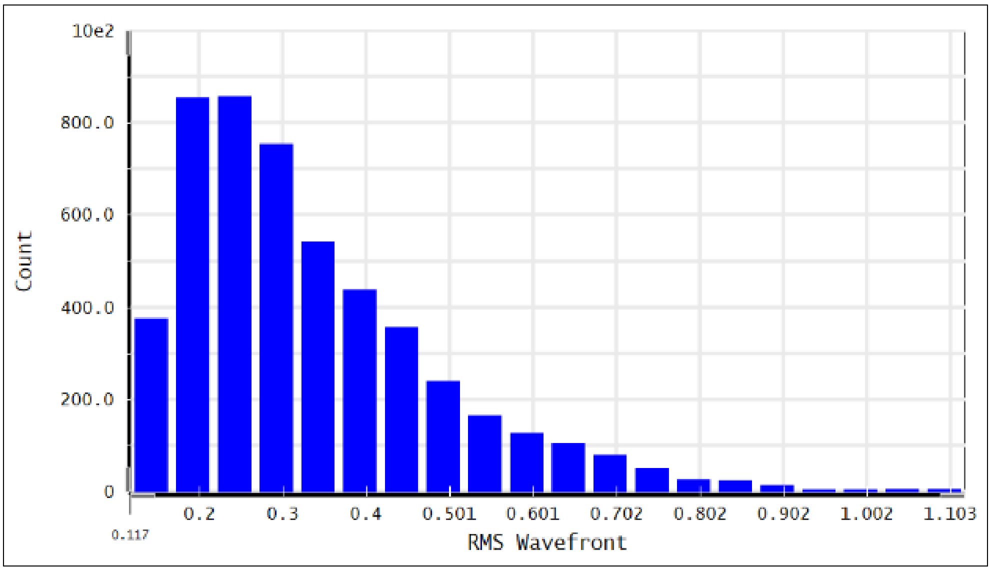
4. Yield rate prediction by Monte Carlo method

Yield rates of the optical system with tolerance sets from this paper and [6] are predicted by using Monte Carlo method with 5000 samples. RMS wavefront error is used as the merit function and the nominal value is 0.1165177. In the simulation for our tolerance sets, paraxial focus is taken as the compensator, but in the simulation for tolerance sets in [6], compensator is not considered. Fig 4 shows the histograms and table 6 lists the yield rates.

From the table 6, we can predict that there are few difference between the performances of the optical systems according to the tolerance sets of [6] and this paer.



(a) result of this paper



(b) result of [6]

Fig 4: Histogram

Table 6: Monte Carlo statistics for my and [6]'s tolerancing model

Yield rate (%)	Tolerance sets of this paper	Tolerance sets of [6]
98	0.73991033	0.71836222
90	0.54248091	0.58190084
80	0.44032398	0.50673762
50	0.29385034	0.38770763
20	0.20618009	0.28251905
10	0.17561026	0.23685390
2	0.14305696	0.17710616

5. Conclusion

In this paper, a method for tolerance allocation of optical systems based on compensator optimization is presented. Sensitivity coefficients of the structural parameters are determined by compensator optimization and tolerance is assigned to solve the minimum cost function problem. In the given example, Cooke triplet is analyzed and the result shows that the frist order and second order cross sensitivity coefficients of the surface decenter and tilt are all zero and the second order sensitiviy coefficents are relatively large. This is coincident with the surface tilt and decenter have the significient effect on the performance of the optical system. The yield rate is predected using Monte Carlo method and through the comparison with the results with [6], it is verified that the our tolerance allocation method that takes into account compensator has looser tolerances than [6] but same perforamce.

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7. Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

8. Disclosure statement

No potential conflict of interest was reported by the authors.

9. Data Availability

The data that support the findings of this study are available within the article.

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