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## A review of methodologies for solving problems in discrete mathematics

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### Abstract

Discrete mathematics (DM) activities, it is said, provide a form of fresh start for pupils and instructors. Pupils who have been blench by conventional education mathematics and instructors whose training has long been standardized may discover opportunities for mathematical inquiry and fascinating, non-routine problem solving in the field of DM. At times, formerly underachievement pupils depict mathematical talent that their instructors were unaware of. To fully exploit these opportunities, it is crucial to understand the forms of thinking that discrete mathematical problems naturally elicit when establishing a program curricula, the purports might comprised routes to desirable numerical thinking processes. This study propose some of these patterns of thought, with an emphasis on the notion of modelling the general in the particular. Additionally, specific observations are made on possible emotional paths and frameworks for pupils.

**Keywords:** Problems solving, heuristics, methodologies

### 1. Introduction

It is frequently stated that the material of discrete mathematics is suited for providing instructors and students with a new perspective on mathematics. Math investigation of non-curricular topics is more simpler here than in many other areas of mathematics. This is true even for less productive school students. However, in order for instructors to fully reap the benefits, they must understand the type of mathematical thinking and reasoning necessary for such projects. The article covers the subject in detail, with an emphasis on modeling the whole scenario using a single example. The author of <sup>[1]</sup> offer a vision for discrete mathematics in American educational institutions in their companion paper, a vision to which they have both bolstered significantly over the recent decade. They view DM a broad term that encompasses multimodal, vertices of graphs, recirculation and recursion, and a variety of various courses as supporting instructors with a new approach to talk about conventional arithmetic topics and a new policy for engaging their pupils in numerical study. They suggest that by gaining expertise in discrete mathematics, instructors may improve their ability to assist children in "thinking critically, solving problems and making decisions using mathematical reasoning and procedures". And they stress, citing <sup>[2]</sup> that "*if discrete mathematics is taught in schools as a collection of facts to be learned and procedures to be used frequently its virtues as a forum for problem solving, reasoning, and experimentation are inevitably destroyed*".

It's enticing to consider that students who have been "put off" by conventional teaching mathematics, as well as educators who have long ago standardized their curriculum, may discover something unique here. Clearly, the distinctive possibilities are less about specific combinatorics formulas and processes, search and sorting algorithms, or graph theorems, and so on and more about the possibilities for engaging, nonroutine problem solving and numerical discovery that DM provides <sup>[3,4]</sup>. We'd want to emphasize how crucial it is to more fully characterize these opportunities, both numerically and intuitively.

### The below points are worth considering

- What particularly desired modes of thought, robust problem-solving procedures, or other critical mathematical abilities do DM circumstances inherently elicit?
- What specific processes or talents are elicited in students? How likely are they to arise in the presence of a problem? Why could we predict the emergence of previously undiscovered numerical talents in some pupils, and what are these skills?
- How can we design students' activities in such a way that they continue to develop the recognized numerical capabilities? Can we achieve this more easily or intuitively with

discrete mathematics than with a identical dedication to conventional arithmetic or geometry?

- d. How can we judge the extent to which pupil’s performance is improved specifically in DM, and in the subject of mathematics in general?

Of course, a little article can only address a portion of these issues. We will concentrate on two points here:

- a. Thinking procedures for numerical problem solving, particularly the process of modeling the general on the specific.
- b. Pupils’ emotional pathways and structures.

**2. Philosophical point of problems**

To contextualize the issue, let us explore a specific and reasonably possibly the best procedural major issue technique. You’re standing on a riverbank with two buckets. One bucket contains precisely three liters of liquid, while the second has approximately five liters. The buckets are not else marked for measurement. How are you going to transport precisely four liters of liquid away from the sea?

It’s also a subject that we have brought up multiple times with kids and youngsters. Additionally, it does not slot neatly into any of the discrete mathematics domains listed above, it has characteristics with a number of those areas’ problem activities. It is "discrete" in the sense that it entails distinct measures that are authorized at any time by the issue conditions. It is sufficiently adventurous for many to find even the numerical interpretation of the issue conditions hard. As with combinatorics’ "counting problems," it requires the problem solver to design a method for keeping track of what has previously been accomplished. As with coloring and "shortest route" issues, it encourages repeated tries that may fail to satisfy the issue criteria. As is the case with many problems in DM, this one is indicative of a potentially underlying structure that, if identified, would make the answer obvious.

A prototype model, as shown in Figure 1, can be used to depict one such challenge diagrammatically, for each entity representing a configuration with a fixed, known volume of liquid in each of the buckets and each arc representing a step of filling a bucket at the river, emptying a bucket, or throwing water from one bucket into another after the latter is packed [5]. In Figure 1, for instance, the ordered pair of integers (0, 2) denotes the arrangement in which the 3-liter bucket is unfilled and the 5-liter bucket contains 2 liters of water.

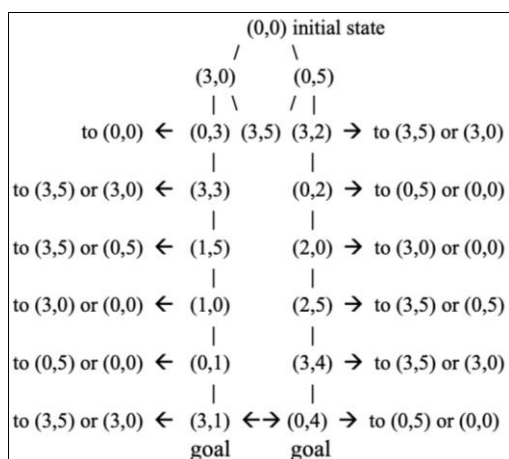


Fig 1: Water jug problem

In reality, this is a network of driven vertices and edges with a chosen apex (node) acting as the "beginning state" and a description of the vertices (nodes) acting as the "target states". As a result, the goal is to identify a path across this network via nearby nodes that ties the initial state to one of the target values. After presenting the graph, we can see that there are pair solution paths that they are connected by congruence but are not identical in terms of pails and movements and that each is just seven steps long. Additionally, we note that there is no way to go wrong while attempting to locate this state-space. That is, if one commence with empty buckets and takes any step, then continues without abandoning or reverting to one of the initial few phases, one will invariably arrive at the desired condition. There are no "blind alleys" in this sequence diagram.

Why, therefore, do those who approach this issue with a strong sense of impasse? What beneficial processes is this situation likely to evoke, and why they are same? Indeed, the problem solver is not looking at a depiction of Figure 1 [6]. The only difficulty is to interpret the challenge statement in a way that makes traditional numerical sense, changing it into one that allows for the accomplishment of something specific and non-arbitrary in the limitations that initially appear to be an implausibility. As a result, some solvers begin their responses with irrelevant thoughts to the issue conditions. Typical first responses include two liters in each bucket without explaining how this arrangement was determined or three and five equal eight, and four equals half of eight, so we will fill each bucket halfway without attending to the condition that the buckets are unmarked for measurement.

As a result, their responses are sensible. Simply because they contradict the stated issue conditions does not mean they are undesirable; in truth, thinking "outside the box" is frequently precisely what non-routine scenarios require. Further interrogation or explanation by the submitter may signal that the issue is not yet resolved: "How do you obtain two liters from each bucket?" Or, "It is impossible to fill the buckets exactly halfway without marking them for measurement." Once it is known that one bucket may be filled to the brim with the knowledge of the volume, and furthermore, that the contents of one bucket may be partially poured into the second bucket until the latter is full, the possibility of an early stalemate emerges. Even if the issue solver is aware of these alternatives that is, if they are conceptually depicted adequately considerable potential entanglements remain. Number of facilitators begin by envisioning themselves pouring water from bucket to bucket, but after three or four pads, they feel stuck and must restart. Some are hesitant or unaware how to make an external written record but the memory burden is significant in the absence of any structured external representation. I have no idea where I was, I’m not going anywhere, and I’m sure I tried this before and it didn’t work are all popular reactions, despite the fact that the potential answer is just one step away from the aim. After a limited series of such attempts, some people give up in frustration. The meticulous record and perseverance of others surmount this barrier.

One reason for this phenomena is the absence of a clear evaluation mechanism by which an intermediate unit, such as (2,5), may be judged "closer" to the aim of owning 4 liters than a state reached three steps earlier, such as (2,1), (3,2). The preponderance of critical thinkers make an

attempt to minimize reverting to previous states, and if this factor is strictly followed, this alone will lead to the desired state. Nevertheless, in the absence of any other signal that one is "coming closer," many persons disregard this criterion and terminate their pursuit. One factor that might explain this trend is the occurrence of two distinct alternate start phases. When a problem solver commences by filling one bucket, she is probably already aware of the danger of selecting the "wrong" option, of making a mistake. Hence more measures completed without achieving the desired result, the more probable it appears as though a "bad" action has already been taken. Prior to completing up to seven stages, the desire to restart may become strong.

Numerous trials that appear to be progressing slowly may produce irritation or embarrassment. On the other side, successful completion of such a job may be thrilling, providing the engaged problem solver with the impression of having made a breakthrough in previously undiscovered mathematical territory. Additionally, this topic provides an obvious opportunity for a student to "reflect" upon completion, noting that the path she had identified is not the sole one to the answer. Few problem solvers appear to accomplish this independently. Additionally, the problem creates hypotheses about the broad characteristics of the problem's conditions and target states, which facilitates solution generation. In practice, few problem solvers spontaneously formulate such hypotheses or enquire about reasonable generalizations.

Clearly, this level of detail allows for the exploration of a large number of discrete mathematics issues. The treatment of a particular subject here is intended to serve as a springboard for understanding how discrete mathematics experiences might serve as a foundation for the generation of significant heuristic processes and effect.

### 3. Creating internal representational systems for mathematical reasoning and issue solving

Consider for a minute the many methods for defining goals, e.g., defining "instructional objectives" for mathematics instruction. One might differentiate two distinct sorts of purposes; or, alternatively, one may see them as two distinct levels at which learning goals might be defined. Standards that are domain-specific and rigorous. We are referring here to the needed skills with widely recognized mathematical concepts, notations, i.e., with commonly used numerical representation systems. These competences contain, but are not limited to, discrete, low-level capabilities; they may (and should) also consist complex methods for addressing a number of pre-established, standard categories of issues, as well as proof methodologies. Additionally, this section includes common mathematical notions that we anticipate pupils to acquire through instruction, i.e., the objective is for our pupils to develop some common cognitive processes that will enable us to communicate quantitatively. By and large, these types of learning outcomes are relatively specialized to the mathematical topic categories in which they are expressed. Illustrative, instructive, and evocative outcomes. We are referring to the required mathematical thinking skills that permit creative and perceptive issue solution but are unrelated to particular harmonic progression abilities. These objectives include the importance of good iconography, including two- and three-dimensional reading comprehension; the potential to construct novel visual and metaphoric expressions in non-standard situations; the

establishment of identity and the potential for structural thinking; and a highly large and nuanced range of methodology problem-solving initiatives<sup>[7, 8]</sup>. Thusly, at this more broad level, we include goals for students' influence<sup>[9]</sup>, not only that they experience the pleasures of mathematics and fulfilment from numerical accomplishment, and also that individuals be able to utilize compelling and relevant interpersonal constructions to numerical pursuits. For instance, in the problem discussed above, a high expectation of success, an expectation of satisfaction from that success, persistence, and an eagerness to interpret dissatisfaction as a signal to be more methodical can all lead to improved process skills, as demonstrated by our talk.

I recently speculated a model for arithmetic learning processes based on five unique components of interpersonal depicted systems<sup>[10, 11]</sup>.

- Oral grammatical systems related to natural language;
- Improvisatory systems, which include visual/geographical, audible, and tactile/ multi - sensory depiction;
- Incorporated proper notational systems of mathematics;
- a planning, surveilling and executive control system that incorporates methodology mechanisms; and
- Effective and useful reflection.

Are all broad biological systems that contain the details of math models, challenges, and analytical approaches.

The domain-specific, structured priorities imply a somewhat more traditional emphasis on learners' obtaining of (c), algebraic topology' formal notational structures, mathematics' metaphors, and strategies for linking multiple within such systems and (a), the fittings vocabulary used to replenish natural language. Although these training is every once in a while ridiculed for being based on rote or meaningless algorithmic processes, it is necessary to keep in mind that symbol-configurations and the stages that connect them do not have to be irrelevant simply because they are stochastic or primarily located within rigorous notational processes. There is an essential difference between abstract mathematical argumentation based on standardized notations (which interact thoughtfully with other stages of mental depiction) and symbol manipulation that is merely decontextualized in the sense that it is removed from substantive, interpretive symbolic circumstances.

All of the improvisatory, logical, and emotional objectives suggest to a heavy emphasis on developing (b), (d) and (e) (e). While these aims are not intrinsically contradictory, because many symbolic systems interact substantially during the cognitive process, it is easy to lose sight of the second set of aims whilst pursuing the first. This is true in the sense that high-stakes, controlled assessments the so-called "achievement tests" on which American educational institutions rely so heavily tend to emphasize more domain-specific, formal objectives by definition.

Thus, while determining how to incorporate DM into the school curriculum, the difficulty of striking a suitable balance between the two types of objectives becomes apparent. Due to the fact that some notations and techniques in DM are powerful tools in their own right for instance, combinatorics, some may choose to see them as legitimate topics such as, in high school algebra.

Eventually, few instructors would advocate for the inclusion of domain-specific methods in solving "water bucket

problems" as conventional learning objectives in the junior high. However, if such obstacles were included on individually administered assessments in order to measure various operational problem solving, we may soon see formulaic strategies for resolving them defined and adhered to a large number of parallel practice tasks. Obviously, the concepts advanced by <sup>[1]</sup>, upon which this essay is based, indicate toward the second class of objectives: those concerned with imagistic, executive, and emotional representation.

To elevate them to a higher level of importance, it becomes critical to emphasize those specific aims in the construction of representational systems for which discrete mathematics provides the greatest options.

#### **4. Methodological procedure: Generalizing the particular model**

A system of planning, supervision, and execution control for mathematics learning that is competent does not consist of easily teachable elements. Instead of another, it tends to develop in the individual as a result of significant critical thinking activities. However, one relevant unit of analysis for such a dynamic system is the intuitive approach. Methodology processes are sophisticated, ill-defined modes of thinking that may be referred to by simple terms such as trial and error, create a picture, or consider an easier challenge.

I and Germain <sup>[12]</sup>, previously proposed that methodological processes encompass four dimensions of cognitive analysis.

- a. Reasons for using a specified task in anticipation of implementation
- b. Domain-specific strategies for implementing the procedures
- c. Contexts and thresholds to which the procedures can be adhered
- d. Directive criteria recommending that the practice be adhered in a particular scenario.

We have an excellent chance to enhance the approach of "models the general on the specific" in all of its manifestations across the discrete mathematical spectrum.

Since numbers are small, discrete mathematics presents relatively straightforward tasks. Combinatorics may be tackled in primary school by beginning with a low-number exploratory methodology and then modeling the general on the particular to derive the answer in a high-number situation. Solving a problem using a graph with a finite number of vertices and edges enables students to uncover a conjecture that can be generalized to a class of much bigger graphs.

When we think mathematically, we should not stop after resolving the first problem; rather, we should assist pupils in developing the ability to pose broader questions. Additionally, the specific can be generalized. Prioritizing a specific issue above a more general one is a qualitative distinction. In the latter case, the intent is more vague and intimidating. Prior to identifying a solution to an issue, specialization must occur. To assist children in formulating generalizing questions, we model the general on the specific. They have contributed significantly to the development of conceptual notions (as well as to what cognitive psychologists refer to as learning transfer).

In addition, we may invite the students to ponder on the bucket problem, asking, Are there any other viable

solutions? Students should be encouraged to produce as many generalizing questions as possible, such as: Can we move any number of liters of water away from the river up to the apparent maximum of 8? Let's find the numbers which can represent the capacity of the buckets. To find a property of two provided numbers, can we solve the problem? Yes. We may take advantage of the issue because of the specific quality. Is there a major difference between a 2-liter bucket and a 9-liter bucket, for example? Do the numbers follow a pattern? How large is the vertex-edge network? Is it a useful generalization if we start with more than two pails? Pupils may not necessarily perform better after developing the capacity to develop and answer arithmetic problems of this kind. Nonetheless, successful methodology development and successful affect are likely to be rewarded. The notion that is derived from answering the question, what constitutes a suitable example for modeling the general on the particular? Is finding the most simple, generic example.

The non-standard, methodical form devised for the issue was quite beneficial. As we speak, we are setting the stage for a heuristic process model to be adapted across various realms. When it comes to modeling a mathematical approach to a broad two-pail problem, we may also model a general strategy for producing non-standard, systematic representations in nonroutine, high-memory burden scenarios. To abstract conditions leads to problem-solving techniques, mathematical equations, and conceptions.

#### **5. Conclusion**

To sum up, discrete mathematics opens you several chances for producing powerful emotions <sup>[5, 13]</sup>. A fresh beginning brings a feeling of optimism, liberation from restraints, and excitement about new and delightful experiences. However, would this promise be kept for students and instructors, particularly those who have previously associated arithmetic with negativity? We must start understanding how discrete mathematical activities might lead to unique, powerful emotional patterns in kids. For example, how can we make efficient use of students' likely feelings of helplessness when presented with a novel problem? We're attempting to elicit feelings of interest and amazement. Others may experience nervousness or anxiety. Individuals may have a sense of inferiority to others in a small-group problem-solving context. Perhaps resistance begins with the statement, I despise these obstacles; these obstacles are ludicrous. In particular, the fact that each event is straightforward to explain and has a lengthy history benefits us. To avoid unnecessary stress, begin by presenting the issue scenario without a goal statement-in the example of the two buckets, just ask, how can you use the bottles? Exploring could be safer, because it takes away the potential for failure see <sup>[5]</sup> Concern alleviation is, of course, an important goal, but it should not be our primary focus. A feeling of security encourages the learner to respond successfully to similar situations in the future: "Here is the problem. What can we do? Let's investigate!"

We further found ways that kids may learn to effectively voice their disappointment. It's possible frustration might stir a want to give up, or frustration can stir a desire to bear with and persist longer in the effort. To assist pupils learn how to effectively work through their anger so that they may "feel good about their displeasure" (perhaps because it suggests the problem is especially interesting). It is feasible

to anticipate and respond to discontent in discrete mathematics training if it is used as an explicit goal. In summary, we should want to cultivate the affect of accomplishment using discrete mathematics-the paths and mechanisms via which recently failed pupils learn to believe.

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