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Roqia Motammahid
Junior Professor, Department of Physics, Kabul Polytechnic University, Kabul, Afghanistan

Corresponding Author:
Roqia Motammahid
Junior Professor, Department of Physics, Kabul Polytechnic University, Kabul, Afghanistan

Compton scattering cross section differential cross section for Compton scattering in three-level QED

Roqia Motammahid

Abstract

Scattering cross section may be defined in nuclear, atomic, and particle physics for collisions of accelerated beams of one type of particles with targets (either stationary or moving) of a second type of particles. The probability for any given reaction to occur is in proportion to its cross section. Thus, specifying the cross section for a given reaction is a proxy for stating the probability that a given scattering process will occur.in this paper. At first i have been considered differential cross section for Compton scattering with regard to lab frame and CM (center of mass) in QED the second, I analyze Compton Scattering cross section in QCD and QMC Theories.

The various cross sections of unpolarized Compton peppering is designed with respect to both t and $\cos\theta\,.|\mu(e^-\gamma \to e^-\gamma)|^2$ which is assessed to first instruction in α_{em} that are used in perturbative QED. By using two diverse methods in which two center of mass and lab frames are worked the Klein - Nishina formula is derived from. Plots of $\frac{d\sigma}{dt}$ and $\frac{d\sigma}{d\cos\theta}$ resemble classical Thomson scattering at low s. At higher energy $\frac{d\sigma}{d\cos\theta}$ which is decreased quickly to π. With the increase of s the overall cross section decreases.

Keywords: Feynman diagram, Klein-Nishina cross section, mass center, lab frame, unpolarized Compton scattering, QCD, QMC, C.P.H theory, QCD Lagrangian

1. Introduction

To calculate the probabilities for relativistic scattering processes, we need to find out the Lorentz- invariant scattering amplitude which connects an initial state containing some particles with well-defined momenta to a final state containing other particles also with well-defined momenta. We make use of a graphical technique popularized by Richard Feynman, each graph- known as a Feynman diagram. The diagram give a pictorial way to represent the contributions to the amplitude. diagram consist of lines representing particles and vertices where particles are created or annihilated, Therefore we can be use Feynman diagram to explain Compton scattering.

While electromagnetic radiation is scattered by the free electrons at rest of the lab reference frame on that time Compton scattering occurs. An electron and a photon of $e^-\gamma \to e^-\gamma$ are the final and initial states. Intrinsic is the cross section which is made by the interaction of the crashing units and let us to calculate the possibility of this final state, and any particular experiment of the independence and luminosity of L. The equivalent scattering events number N is connected to the cross section by:

$$N = L.\sigma \tag{1.1}$$

The following is the most common formula of insignificant cross section of a two particle collision which is given by [1,4,10].

$$d\sigma = \frac{1}{2E_A 2E_B|v_A - v_B|}|M\ (p_A, p_B \to \{pf\})|^2 d\Pi_n \tag{1.2}$$

This is the form of phase space integral over the final states $\int d\Pi_n =$

$$\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)(2\pi)^4\delta^{(4)}\left(\sum p_f - p_A - p_B\right) \tag{1.3}$$

The Lorentz invariant is the last two factors in (1.2), in which the first is invariant under co-linear boosts. Lorentz transformations,

$$\mathcal{X}^\mu \to (\mathcal{X}')^\mu = \Lambda^\mu{}_\nu \mathcal{X}^\nu \tag{1.4}$$

Here $\mathbf{\Lambda}^{\mu}{}_{\nu}$ satisfies. $\mathbf{\Lambda}^{\mu}{}_{\sigma}\ \eta^{\sigma\tau}\ \mathbf{\Lambda}^{\nu}{}_{\tau} = \eta^{\mu\nu}$ (1.5)

The square matrix element $|M|^2$ may be calculated using the Feynman rules of quantum electrodynamics (QED) without any reference to a particular frame of reference. The Feynman rules tell us how to go from a diagram to the corresponding matrix element (or amplitude) which is necessary to calculate σ and Γ. this calculation, evaluating the remaining factors in both the centers of the mass and lab frames will be undertaken. This type of evaluation let us to compute the various cross sections with the respect of the both: $\frac{d\sigma}{dcos\theta}$ is the scattering angle in the lab frame and $\frac{d\sigma}{dt}$ is the squares momentum of transfer among the initial-state and final-state photons. [1, 5].

2. Calculating the square S-matrix element

Refer to Feynman's rules and diagrams for explanation, The Feynman rules tell us how to go from a diagram to the corresponding matrix element (or amplitude) which is necessary to calculate σ and Γ.

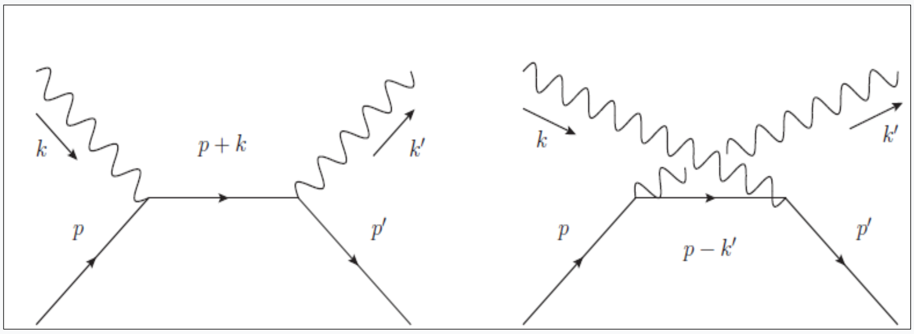

Fig 1: Compton scattering, Feynman diagrams. Time goes from the left to right

Initial and final states representative of Compton scattering in a zero loop (tree) level are the two possible Feynman diagram. These two levels are shown in figure (1). Following is the converse fermion flow and QED applying of Feynman rules to each diagram, the immediate corresponding transition amplitudes should be written as M1 and M2:

$$i\,M_1 = u^{-\acute{s}}(\acute{p})(-ie\gamma^{\mu})\epsilon_{\mu}^{*\acute{r}}(\acute{k})\left(\frac{i(p+k+m)}{(p+k)^2-m^2}\right)(-ie\gamma^{\nu})\epsilon_{\nu}^{r}(k)u^{s}(p) \tag{2.1}$$

$$i\,M_2 = u^{-\acute{s}}(\acute{p})(-ie\gamma^{\nu})\epsilon_{\nu}^{r}(k)\left(\frac{i(p-k+m)}{(p-k)^2-m^2}\right)(-ie\gamma^{\mu})\epsilon_{\mu}^{*r}(\acute{k})u^{s}(p) \tag{2.2}$$

Where the spinor $u^{(s)}(p)$ represents the incoming electron and $\in^{\mu}$ is the polarization vector describing our incident photon. The probability amplitude for a transition of a quantum system (between asymptotically free states) from the initial state $|i\rangle$ to the final state $|\,f\,\rangle$ is given by the matrix element$S_{fi} = \langle f|S|i\rangle$, where S is the S-matrix. In the QFT canonical quantization, in the Wick expansion of the scattering matrix element $\langle f|S|i\rangle$ [3] the scales are the terms. So, the entire conversion largeness is equal to the amount of these expression, [1, 5, 7].

$$M = M_1 + M_2 \tag{2.3}$$

No relative minus sign is existed among the two terms in line for the identical fermion flow in both diagrams accordance with the Fermi- Dirac statistics. To add in this, if there is no scattering occurrence, there is not interested corresponded to the process.

So, there is: $\langle f|S-1|i\rangle = iM(2\pi)^4\delta^4(P_f - P_I)$ (2.4)

All S-matrix elements shares the momentum conserving delta function, therefore it is engaged in to the common formulation for a cross section (1.2). for calculated of this formula we need $|M|^2$ The formula can be taken by:

$$|M|^2 = (M_1 + M_2)(M_1^* + M_2^*) = |M_1|^2 + |M_2|^2 + 2R(M_1M_2^*) \tag{2.5}$$

The whole square matrix element is consequently alike to the amount of three terms which can be calculated unconnectedly: in which the two terms are simply equal with the square of each amplitude and a third 'interference' term between the diagrams. It may be started with evaluation of $|M_1|^2$ before returning to the latter two terms. For Calculating $|M_1|^2$ We may simplify the expression forM_1 (2.1). First, by rearranging commuting terms:

$$iM_1 = -ie^2\acute{\epsilon}_{\mu}^{*}\epsilon_{\nu}\bar{\acute{u}}\left(\frac{\gamma^{\mu}(\not{p}+\not{k}+\not{m})\gamma^{\nu}}{(p+k)^2-m^2}\right)u \tag{2.6}$$

The denominator may then be rewritten as follows:

$$(p+k)^2 - m^2 = p^2 + p \cdot k + k \cdot p + k^2 - m^2$$

$$= 2p \cdot k \qquad (2\text{-}7)$$

$p^2 = m^2$ and $k^2 = 0$. By using of the following formula the numerator also can be simplified

$$(p+m)\gamma^\nu u = \left(\gamma^\mu \gamma^\nu p_\mu + \gamma^\nu m\right)u = (2p^\nu - \gamma^\nu p + \gamma^\nu m)u = 2p^\nu u \qquad (2\text{-}8)$$

In which equality of the second follows from the Dirac algebra $[\gamma^\mu, \gamma^\nu] = 2\eta^{\mu\nu}$and third quality flows from the calculation of signal for positive frequency results of the free Dirac field $(p-m)u = 0$. if we find the complex conjugate M_1^*, then we make the spin or keys clear in which we are able to reorganize $|M_1|^2$. the result of summing over polarization state, trace algebra and Lorentz Mandelstam variables, which can be taken directly from 4- motion maintenance $p + k = p' + k'$ [5,12].

$$\begin{aligned}
s &= (p+k)^2 = 2p.k + m^2 \\
&= (p'+k')^2 = -2p'.k' + m^2 \\
t &= (p'-p)^2 = -2p.p' + 2m^2 \\
&= (k'-k)^2 = -2k.k' \\
u &= (p'-k)^2 = -2k.p' + m^2 \\
&= (k'-p)^2 = -2p.k' + m^2
\end{aligned} \qquad (2\text{-}9)$$

These also yield the relation $s + u + t = 2m^2$. Considering the Dirac algebra, Dirac free field, also specific spin and polarization states for the electrons and photons we receive

$$\overline{|M_1|^2} = 2e^4 \left[\frac{4m^4}{(s-m^2)} + \frac{2m^2}{(s-m^2)} - \frac{(u-m^2)}{(s-m^2)}\right] \qquad (2\text{-}10)$$

We may simplify $M2$ in the same way as 1 [5].

$$\overline{|M_2|^2} = 2e^4 \left[\frac{4m^4}{(u-m^2)} + \frac{2m^2}{(u-m^2)} - \frac{(s-m^2)}{(u-m^2)}\right] \qquad (2\text{-}11)$$

Now, to calculate the interference term $2M_1M_2^*$we must chose the complex conjugate $-iM_2^*$ and substituting the Mandelstam variables, gives us:

$$\overline{2M_1M_2^*} = 2e^2 \left[\frac{8m^4}{(s-m^2)\ \ (u-m^2)} + \frac{2m^2}{(u-m^2)} - \frac{2m^2}{(s-m^2)}\right] \qquad (2\text{-}12)$$

Togetherness of the whole terms at the end give us:

$$\overline{|M|^2} = 2e^4 \left[4m^4 \left(\frac{1}{s-m^2} + \frac{1}{u-m^2}\right)^2 + 4m^4 \left(\frac{1}{s-m^2} + \frac{1}{u-m^2}\right)^2 - \frac{u-m^2}{s-m^2} - \frac{s-m^2}{u-m^2}\right] \qquad (2\text{-}13)$$

We consider the bellow ideas for calculating of the cross section from this:

3. Center of mass frame

Mass (CM) frame center, based on the 4 momenta diagram of the paticles can be written as follow:

$$\begin{aligned}
k &= (\omega^*.0.0.\omega^*) & \acute{k} &= (\omega^*.\omega^* \sin\theta^*.0.\omega^* \cos\theta^*) \\
p &= (E.0.0.-\omega^*) & \acute{p} &= (E.-\omega^* \sin\theta^*.0.-\omega^* \cos\theta^*)
\end{aligned} \qquad (3\text{-}1)$$

According to the above direction ω^* and θ^* are the photon energy and scattering angle in the CM frame. This makes the Mandel stem variables [1,2]:

$$\begin{aligned}
s &= 2\omega^*(E+\omega^*) + m^2 \\
t &= 2\omega^*(cos\theta^* - 1) \\
u &= -2\omega^*(E + \omega^* cos\theta^*) + m^2
\end{aligned} \qquad (3\text{-}2)$$

Therefore, (1.2) becomes:

$$d\sigma = \frac{1}{2E2\omega^*\left|1-\frac{\omega^*}{E}\right|} \overline{|M|^2} d\Pi_2 \qquad (3\text{-}3)$$

In terms of frame invariant variables

$$d\sigma = \frac{1}{2(s-m^2)}\overline{|M|^2}d\Pi_2 \quad (3\text{-}4)$$

In line for the last state point space integral in the CM frame the bellow idea can be get

$$\int d\Pi_2 = \iiint \frac{d^3\acute{p}}{(2\pi)^3}\frac{1}{2E_{\acute{p}}}\iiint \frac{d^3\acute{k}}{(2\pi)^3}\frac{1}{2E_{\acute{k}}}(2\pi)^4\delta^{(4)}(p+k-\acute{p}|-\acute{k}) \quad (3\text{-}5)$$

Concluded all 3 components integration of $\acute{p}$, give $p = -k\ and$ arranges$\acute{}p = -\acute{k}$. Also, by the change of the variable to sphere-shaped polar matches in phase space[5] we have

$$\int d\Pi_2 = \iiint \frac{d\acute{k}\acute{k}^2 d\cos\theta^* d\phi^*}{(2\pi)^2 4E_{\acute{p}}E_{\acute{k}}}\delta\left(E+\omega^* - E_{\acute{p}}(\acute{k}) - E_{\acute{k}}(\acute{k})\right) \quad (3\text{-}6)$$

There is a symmetric impact about ϕ, so we take the $2\pi\ factor$. So, by having this factor and impact we have an essential over momentum, but here the Dirac delta function is in terms of energies, in which functions of momentum is by the dispersion relation.so if $f(k') = E + \omega^* - E_{p'}(k') - E_{k'}(k')$, we must evaluate

$$\int d\Pi_2 = \iint \frac{dk'k'^2 d\cos\theta^*}{8\pi E_{p'}E_{k'}}\frac{\delta(k'-k'_0)}{\left|\frac{\partial f(k'_0)}{\partial k'}\right|}$$

$$= \int \frac{k'^2 d\cos\theta^*}{8\pi}\left(\frac{k'}{E_{p'}+E_{k'}}\right) \quad (3\text{-}7)$$

According to the above formula $f(k'_0) = 0.$ the exact value of $k'_0$is unnecessary' but togetherness on this distribution imposes energy preservation $E + \omega^* = E_{p'} + E_{k'}$.hence:

$$\int d\Pi_2 = \int d\cos\theta^* \frac{\omega^*}{8\pi(E+\omega^*)} \quad (3\text{-}8)$$

It can be also calculate in the CM frame of $dt = 2\omega^{*'} d\cos\theta^*$. Thus

$$\int d\Pi_2 = \int \frac{dt}{16\pi\omega^*(E+\omega^*)} = \int \frac{dt}{8\pi(s-m^2)} \quad (3\text{-}9)$$

The above idea for the cross section gives us a clearly invariant formula of the Lorentz.

$$\frac{d\sigma}{dt} = \frac{1}{16\pi(s-m^2)}\overline{|M|^2} \quad (3\text{-}10)$$

For more explanation, it's necessary to consider lab frame:
In the lab frame the 4-momenta of the particles may be written as follow because the electron is at the rest

$$k = (\omega.0.0.\omega) \qquad k' = (\omega.\omega\sin\theta.0.\omega\cos\theta)$$
$$p = (m.0) \qquad \acute{p} = (E.-\acute{p}) \quad (3\text{-}11)$$

The bellow formula makes the Mandelstam variables in the lab frame:

$$s = 2m\omega + m^2$$
$$t = 2\omega\acute{\omega}(\cos\theta - 1)$$
$$u = -2m\acute{\omega} + m^2 \quad (3\text{-}12)$$

In the bellow frame $\acute{\omega}$ and $\theta\ both$ are not independent variables. For finding this dependence, we work from Compton formula. Keeping 4-momentum $p + k = \acute{p} + \acute{k}$ once again

$$\acute{p}^2 = \left(p + k - \acute{k}\right)^2 = p^2 + 2p.\left(k - \acute{k}\right) - 2k.\acute{k} + k^2 + \acute{k}^2 \quad (3\text{-}13)$$

$$m^2 = m^2 + 2m(\omega - \acute{\omega}) + 2\omega\acute{\omega}(\cos\theta - 1) \quad (3\text{-}14)$$

Thus,

$$\frac{1}{\acute{\omega}} - \frac{1}{\omega} = \frac{1}{m}(1 - \cos\theta) \quad (3\text{-}15)$$

Rearranging for$\acute{\omega}$

$$\acute{\omega} = \frac{\omega}{1+\frac{\omega}{m}(1-\cos\theta)} \tag{3-16}$$

The above information permits us to define t only in terms of $\cos\theta$:

$$t = \frac{2\omega^2(1-\cos\theta)}{1+\frac{\omega}{m}(1-\cos\theta)} \tag{3-17}$$

$$\frac{dt}{d\cos\theta} = \frac{2\omega^2}{(1+\frac{\omega}{m}(1-\cos\theta))^2} = 2\acute{\omega}^2 \tag{3-18}$$

With respect to the angle in the lab frame we can find the different cross section:

$$\frac{d\sigma}{d\cos\theta} = \frac{d\sigma}{dt}\frac{dt}{d\cos\theta} = 2\acute{\omega}^2\frac{1}{16\pi(2m\omega)^2}\overline{|M|^2} \tag{3-19}$$

For expressing $\overline{|M|^2}$ according to the lab frame variables we substitute (3-12) and (3-16) in to (2-13) to get

$$\overline{|M|^2} = 2e^4\left[4m^4\left(\frac{\cos\theta-1}{2m^2}\right)^2 + 4m^2\left(\frac{\cos\theta-1}{2m^2}\right) + \frac{\acute{\omega}}{\omega} + \frac{\omega}{\acute{\omega}}\right]$$
$$= 2e^4\left[\frac{\acute{\omega}}{\omega} + \frac{\omega}{\acute{\omega}} - \sin^2\theta\right] \tag{3-20}$$

Which finally produces the Klein-Nishina formula

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m^2}\left(\frac{\acute{\omega}}{\omega}\right)^2\left[\frac{\acute{\omega}}{\omega} + \frac{\omega}{\acute{\omega}} - \sin^2\theta\right] \tag{3-21}$$

Still via phase space integral in the lab frame the same result can be obtained [1,4,5].

4. Discussion

Graph is about symmetric$\cos\theta = 0 in\ the\ low\ energy\ limt\ of\ the\ reference\ frame$. Forward and back scattering are alike of each other.

Limit of $\omega \to 0, \frac{\acute{\omega}}{\omega} \to 1$. Therefore, (3-21) converts: $\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m^2}(1+\cos^2\theta)$ (4-1)

The above is the Thomson scattering formula and is taken from the limit of classical electromagnetism. With the unchanging of $\theta = 0$ scattering event a higher energy scattering probability decreases. With the high s limitation backward scattering probability become low and is constant with the angle, and θ becomes smaller with the rapid increase of forward scattering. Figure 2 also shows a likeness of the classical scattering of electromagnetic radiation for mass energy low center, and it can be shown with the center of mass energy increases and by this work the whole cross section decrease. At high S the degree of different cross section is bigger for higher$|t|$ deceasing quickly as t approaches 0.

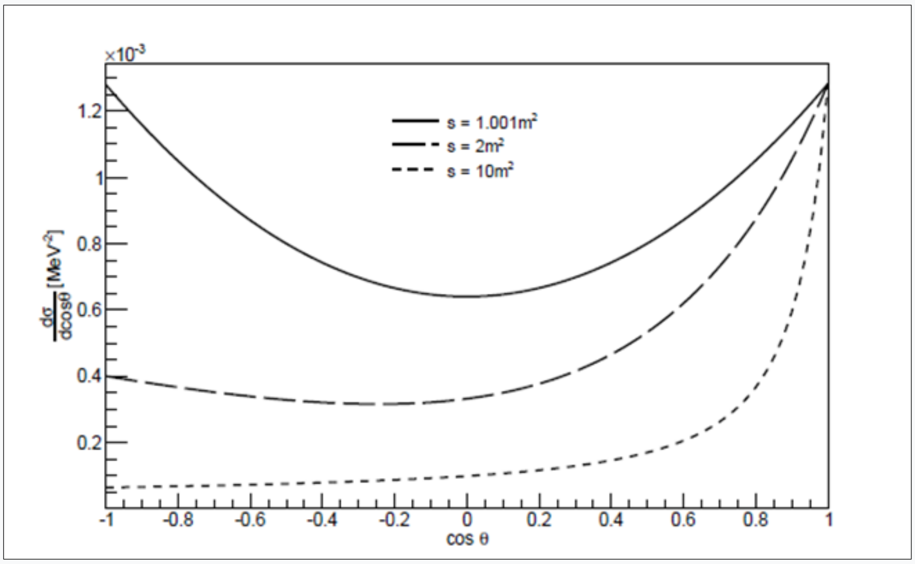

Fig 2: This shows the differences of Cos θ for three dissimilar center of mass dynamisms.

For scattering and cross section, there is also a definition of the whole decay rate by adding of the whole momenta; and is divided by the symmetry factor.

$$\Gamma = \frac{1}{s}\int d\Gamma \tag{4-2}$$

S is the symmetry factor [1, 5].

5. Photon analysis and its importance

Now we analyze Compton scattering cross section in QCD and QMC Theories, to achieve this destination, we must consider the analysis of photon. the photon is a type of elementary particle. It is the quantum of the electromagnetic field including electromagnetic radiation such as light, and the force carrier for the electromagnetic force. Photons are massless, so they always move at the speed of light in vacuum, $3 * 10^8 \frac{m}{sec}$. The photon belongs to the class of bosons. like all elementary particles, photons are currently best explained by quantum mechanics, their behavior featuring properties of both wave and particle.

At first, the photon was regarded as structure less as the scale of available energies increased, it was found that through an interaction with a coulomb field, the photon could materialize as pairs of electrons $\gamma = e^+e^-$. In reality the photon has an internal structure which is very similar to that of hadrons, except that it occurs with a probability only of order $a \sim \frac{1}{173}$, [1, 2].

In quantum field theory, the electromagnetic field couples to all particles carrying the electromagnetic current, and thus a photon can fluctuate into virtual states of remarkable complexity. At high energies, the fluctuation of a photon into a folk state of particles of total invariant mass M can persist over a time of order $\tau = \frac{2E_\gamma}{M^2}$ until the virtual state is materialized by a collision or annihilation with another system.

Considering the soft processes involving photon, in vector Dominance Model (VDM) and Generalized (GVDM), as t increases it is very unlikely that the process remains elastic. The inelastic production starts to dominate, nevertheless one can still find in the photon-hadron scattering a similarity to the pure hadron-hadron collision. In both cases, for example, in the hard inclusive processes, the quark and gluon degrees of freedom come into the game. This is expected since by similar reasoning as above, the transition $\gamma = qq^-$ Which may occur in a color field of hadronic constituents, should be treated as a signal of the quark constituent in the photon.

Hard hadronic processes involving partonic constituents of the photon can be described in Quantum Chromodynamics (QCD) due to smallness of the corresponding coupling constant s a$a_s(Q^2)$, with Q^2 being the hard scale. Contrary to the structure of hadrons, the structure functions for the photon can be calculated in the Parton Model and already at (Born) level the scaling violation appears. The all-order logarithmic Q^2 dependence of the partonic densities in the photon can in principle be calculated in QCD in a form of the asymptotic solutions, without the extra input at some scale, needed for hadrons [1, 5, 2].

The cross section for the process involving the interaction of the photon with elementary charged particles, can be presented symbolically as a series in the coupling constant $a = \frac{e^2}{4\pi}$ $\sigma \sim a + a^2 + \cdots$

For small coupling constant, one can approximate the cross section by the first, or by first few terms in the above expansion. However, for some inclusive processes involving a large energy scale, the expansion parameter may be different, there may appear large logarithms which should then be summed up to all orders.

The study of photons is very important in C.P.H theory (Creative Particles of Higgs Theory), it has been attempted to scrutinize the interface between classical mechanics, relativity and quantum mechanics, through a novel approach to the established physical events. To analyze the Doppler Effect and red (or blue) shift of the gravitational, still for efforts and attempts to recognize and explain the structure of photon, it is necessary to analyze the C.P.H theory. C.P.H theory has formed based on a definition from the structure of photon. Some particles like the photon move only with the speed of light, in all inertial reference frames. Let’s call these kinds of particles the NR-particles or Never at Rest condition particles. In C.P.H theory, description the structure of photon is based on the behavior of photons in the gravitational field. In effect gravitons behave as, if they have electric and magnetic field effect. These are referred to as negative color charge, positive color charge and magnetic color. From this, it can be shown that a photon is made of color charges and magnetic colors. The attention inside the photon structure is very useful and important for understanding QCD phenomena. Equivalence relation of mass-energy conception is beyond converting matter into energy and vice versa. Because what is at the core of the interaction, between quarks in the proton structure occurs is the logical result of interaction between the SQE_s
(or $\lhd, \rhd$) in photon structure. When you convert energy into matter, the properties of interaction between SQE_s are also transferred from the photon to particle-antiparticle and vice versa. According to Compton Effect and gravitational blue-shift, energy of a photon can decrease or increase without changing in its physical properties (except its energy and frequency). It means that whatever is increased to the energy of photon, it has the same total properties of photon (properties of electromagnetic energy). We can also use the Monte Carlo technique, to investigate the accuracy of cavity theory for photons, considering energy range [5, 2].

6. Compton scattering cross section analysis in QCD and QMC

Now we can discuss the Compton scattering cross section, in QCD and QMC Theories.

The forward (small scattering angle) Compton scattering cross section is then a valuable method to measure the QED coupling. but for strong interactions, we can use quantum chromodynamics (QCD). the major difference between QED and QCD is that the gluons are self-interacting because they also carry color charge (unlike the charge-neutral photon).

Quantum chromodynamics (QCD) is the theory of strong interactions. It is formulated in terms of elementary fields (quarks and gluons), whose interactions obey the principles of a relativistic QFT, with a non-abelian gauge invariance SU (3). The emergence of QCD as theory of strong interactions could be reviewed historically, analyzing the various experimental data and the theoretical ideas available in the years 1960-1973.
We know that hadrons are mad of quarks, that quarks are spin-1/2, color-triplet fermions, interacting via the exchange of an octet of spin-1 gluons, the concept of running couplings, asymptotic freedom and of confinement, and some familiarity with the fundamental ideas and formalism of QED: Feynman rules, renormalization, gauge invariance. This helps us to understand the content, also it would be better to understand the contents of this article, we have an analysis of QCD Lagrangian.
The particles which carry color charge are
Spin-1/2: six families of quarks (up, charge and top with electric charge +2/3; down, strange and bottom with electric charge -1/3)

For each flavor, there are $N_c = 3$ of these.
Spin-1: $8 = (N_c^2 - 1)$ massless gluons.
The QCD Lagrangian for a quark of mass m is

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a\mu\nu} F^{a}_{\mu\nu} + \psi_i\left(iD_{ij} - m\delta_{ij}\right)\psi_j, \tag{5-1}$$

With $D^{\mu}_{ij} = \partial^{\mu}\delta_{ij} + ig_s t^{a}_{ij} A^{a\mu}$, (5-2)

$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{v} - \partial_{v}A^{a}_{\mu} + g_s f^{abc} A^{b}_{\mu} A^{c}_{v} \tag{5-3}$$

The a, i and j indices are gauge group indices which are discussed further below. The QCD Lagrangian $\mathcal{L}_{QCD}$ is invariant under the infinitesimal 'gauge' transformations

$$\psi_i(x) \rightarrow \left(\delta_{ij} - ig_s\theta^{a}(x)t^{a}_{ij}\right)\psi(x) \tag{5-4}$$

$$A^{a}_{\mu}(x) \rightarrow A^{a}_{\mu}(x) + D^{ab}_{\mu}\theta^{b}(x) \tag{5-5}$$

Where D^{ab}_{μ} is the covariant derivative in the 'adjoint' representation, the one under which the gluon fields transforms under SU (3), as opposed to the 'fundamental' representation, which rules the transformation of quark fields.
Notice that the gauge transformation for A^{a}_{μ} involves the strong coupling g_s:

$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\theta^{a} + g_s f^{abc}\theta^{b} A^{c}_{\mu} = A^{a}_{\mu} + \partial^{a}_{\mu}\theta^{a} + \mathcal{O}(g_s) \tag{5-6}$$

And only at lowest order in g_s does it reduce to the analogous transformation for QED.
As in QED, in order to quantize the QCD Lagrangian, we need to introduce a 'gauge fixing' term, for instance

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}\left(\partial_{\mu}A^{\mu}_{a}\right)^{2}. \tag{5-7}$$

Returning to the Lagrangian, in QCD $F^{a}_{\mu\nu}$ has an extra term compared to QED, as required by gauge invariance. (Technically this term is present for QED too, but QED is an "Abelian" gauge theory which means that the structure constants are zero). the presence of the non-commuting color matrices illustrates that SU (3) is a non-Abelian gauge group. We can see the effect of this by studying the QCD equivalent of photon pair Production, $q(p)\bar{q}(\acute{p}) \rightarrow g(k)g(\acute{k})$. In QED, the matrix element squared for this process can be obtained from that of Compton scattering via crossing. Considering the Feynman diagrams for this process and in covariant gauges, like the Feynman gauge we understand the fields which is called the ghost fields. They can propagate and couple to gluons, but never appear in physical final states. and about the beta function, we can understand, that the beta function of QCD, $\beta_{QCD}(\alpha_s) = -\frac{21}{12\pi}\alpha_s^2$, is negative when α_sis small means that the QCD coupling decreases with energy. This property is known as asymptotic freedom, and is crucial to be able to compute hadronic cross sections in terms of quarks and gluons.
Compton scattering in its various forms provides a unique tool for studying many aspects of hadronic structure by probing it with two electromagnetic currents. For real Compton scattering (RCS) in the hard scattering regime, where all Mandelstam variables $s, -t$ and $-u$ are larger than the A^2_{QCD} scale, the short-distance dominance is secured by the presence of a large momentum transfer. In this regime, RCS probes the fundamental quark-gluon degrees of freedom of quantum chromodynamics (QCD), providing important information for the tomographic imaging of the nucleon. The only data for RCS in the hard scattering regime were obtained 25 years ago by the pioneering Cornell experiment. The cross section $d\sigma/dt$ at fixed θ_{cm} was found to scale with $1/s^n$ with $n \approx 6$, exactly as predicted by perturbative QCD, in which the reaction is mediated by the exchange of two hard gluons. Nevertheless, the experimental cross section was at least 10 times larger than those predicted by perturbative QCD. more recently, calculations of RCS have been performed within a handbag dominance model, in which the

external photons couple to a single quark, which couples to the spectator particles through generalized Parton distributions (GPDs). These calculations are rather close to the Cornell cross section data.

Compton scattering is simulated by all general purpose Monte Carlo systems that model the transport of photons and other particles in matter. Nevertheless, an objective, quantitative evaluation of the physical accuracy of Compton scattering simulation models is not yet documented in the literature. The validation of Compton scattering models implies their comparison with experimental data [7, 11]. In Monte Carlo systems we use REFACTORED SOFTWARE. The software design has been refactored based on a sharp domain decomposition, which identified total cross section calculation and final state generation as two distinct entities of the problem domain. A policy-based class design, has been adopted; it ensures flexibility at endowing the Compton scattering process with multiple behaviors based on a variety of alternative modeling approaches, while the intrinsic simplicity, restricted responsibilities, and minimized dependencies of policy classes facilitate the testing of the software both in the processes of verification and of validation. Several programs have implemented the sec-tor decomposition algorithm for the numerical evaluation of Feynman loop integrals. Normally they use Monte Carlo (MC) integration methods, which have been widely used in high energy physics research [5, 7].

7. Summary

It is shown that what is the difference for cross section analysis between QED, QCD and QMC theories, and what is the importance of center of mass frame, square S-matrix element, photon structure for analysis scattering process in these theories. I hope it has provided some insight in the clarity and analysis for Compton scattering descriptions in different theories.

8. References

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