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Mathematics in India during middle period (400 AD - 1200 AD)

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Abstract
In India, Mathematics has been in use since Harappa civilization. There have been found various evidences of usage of number system, weights and measures from the sites of this civilization. After this civilization, there has been a continuous progress in the field of Mathematics in ancient and medieval period. Introducing of decimal system, usage of nuber system, calculating the value of Pie are some of the prominent inventions done by Indian mathematicians. Present work traces the usage of Mathematics during Harappa period and show cases some of the remarkable work of Indian mathematicians during middle period.

Keywords: Middle period, Aryabhatt, Varahmihira, Brahamagupta, Varahmihira

1. Introduction
In almost all the civilizations, the earliest form of Mathematics appears in the form of number system. The earliest sign of use of Mathematics in India come from Harappa civilization around 3000 BC. This civilization was urban in nature and was near the Indus river. During the excavation of the area around Indus river, the evidences of usage of basic Mathematics were found. The decimal system on which the foundation of number system is built was there in place during Harappa period. The people of this civilization were generally concerned with weights and measures. A uniform system of weights and measures was in place at that time. There were also found some tools of measuring of length. An accurate decimal ruler was found. The maximum error of this ruler is 0.005 at a length of 1.32 inches. This is an example of the precision with which those people were familiar. This ruler is known as Mohenjodaro ruler and the length of it is known as the Indus inch. One of the finding of excavation was the bronze rod which marked 0.367 inches. This shows that another scale also was in place at that time. The accuracy of this scale was also remarkable. The ruins of the buildings of this period points to the accuracy of length of measurement during their construction. Based on the evidence of carving at the wall, it was also found that the Harappa people could draw intersecting and concentric circles and triangles. One of the interesting thing which came out was that the decorative design consisting of circles were found at the pictures of bullock carts. A metallic band was found wrapped around the rim of bullock cart which is an indication that Harappa people had knowledge of the ratio of the length of the circumference and the diameter of the circle. This shows that they had known the value of π which is remarkable.

The history of Mathematics in India can be divided into different parts based on different periods *viz* Ancient Period (before 2500 BC), Pre Middle Period (500 BC – 400 AD), Middle Period (400 AD – 1200 AD), Later Middle Period (1200 AD – 1800 AD) and Current Period (after 1800 AD). The decimal system on which the foundation of number system is built was there in place during Harappa Period. The example of this was the bronze rod marked 0.367 inches. It shows that precision was there at that time. In Vedic period around 1000 BC, Mathematics was mainly related to ritual activities. An important book *viz Vedang Jyotish* was written during this period. It mentioned the importance of Ganit (Mathematics) as –

"Just as branches of a peacock and jewel-stone of a snake are placed at the highest place of body (forehead), similarly position of Ganit is highest in all the branches of *Vedah*and *Shastras*"

In later Vedic period from 1000 BC to 500 BC, there happened development of Mathematics in India. Arithmetic operations were explained in the Narad Vishnu Purana by Ved Vyasa. Some of the examples of geometry are mentioned in Sulva-Sutras of Baudhayana and Apasthamba. Baudhyana's sutra contains knowledge of shapes of basic geometrical objects like rectangle and square. It also mentions how to convert a geometrical shape to another shape.

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It is said that the Greek Mathematician and Philosopher Pythagoras who lived in 600 BC learnt basic geometry from Sulva Sutras itself. In the period from 500 BC to 400 AD, there have not been much literature available about development of Mathematics in this era. Although some of the books like Surya Siddhanta, VaychaliGanit and GanitAnoyog were written in this period. The middle period from 400 AD to 1200 AD is known as Golden Period of Mathematics. There has been much development of Mathematics in this period. There has been tremendous growth of trade and commerce in this period which require understanding of concepts of simple and compound interest as there was frequent lending and borrowing. Concepts of Arithmetic and Geometric series were developed in this period only. Great mathematicians like Aryabhatt, Bhaskaracharya, Brahmgupt, Mahaveeracharya contributed a lot to mathematics. It is due to their efforts, that the different branches of Mathematics took clear and broad shape. The principles and methods which were in the form of sutras in vedas were introduced to masses by these mathematicians. Work of some of the Mathematicians is included in this work.

2. Aryabhatt (476 AD – 550 AD)
He has written a book titled *Aryabhattiya*. This book contains 332 *Shlokas* and is divided into 4 sections in which Aryabhatt has described the fundamental principles of Mathematics. In the first two sections, the description of Mathematics is given and in last two sections, *Jyotish* (Astrology) is explained. He was the first to find that the Earth revolves around Sun. He found that it is the Sun which is stationary and not the Earth. He refuted the common belief that Earth is stationary. He also calculated the value of Pie correct upto four places of decimal. He found the solution of the equation $ax+b=cy$, where a, b & c are unknown integers, using kuttaka method. He calculated the sum of an Arithmetic Progression. He also worked on finding square and cubic root of a number with the use of decimal number system. He developed algorithm to formulate table of Sines. He was a great astronomer of that time. One of his belief was that the orbits of planets are ellipses which was incredible at that time. Later on, his belief was established by German Astronomer Johannes Kepler. Aryabhatt was successful in explaining the causes of Sun and Lunar eclipses. He also believed that the Moon and planets do not have their own lights rather they shine by the reflection of sunlight from their surfaces. He was the first to use trigonometry to study spherical geometry.

3. Varahamihira (505 AD – 587 AD)
His most famous work is *Pancasiddhantika* (Five Astronomical Canons). It was written in 575 AD. It gives insight on astronomy from mathematical point of view. It is considered as one of the most important source of Hindu astronomy. The place-value number system has been explained in this work with the help of number of examples. He improved upon the Sine tables given by Aryabhatt and discovered trigonometric formulae. Algebraic properties of negative numbers and zeros were defined by him. He also wrote a treatise *Romaka-Sidhhanta* which is based on the theory of motions of Moon and Sun as described by Greek Mathematicians in 1st century AD. It is also said that he predicted the possibility of water on planet Mars although no conclusive evidence has been found on this.

4. Brahmagupta (598 AD- 670 AD)
He wrote a commentary on opening of the universe in his important work *Brahmasphutasiddhanta*. This is considered an important work on Astronomy and Mathematics. He defined zero as the resultant number when a number is subtracted from itself. He gave properties of zeros as "*When zero is added to a number or subtracted from a number, the number remains unchanged; and a number multiplied by zero becomes zero*". He described arithmetic rules with the help of fortunes (positive numbers) and debts (negative numbers) e.g. "*A debt minus zero is debt*", "*A fortune minus zero is fortune*".He made efforts to extend the concept of zero. He tried to find if division of a number from zero is possible. Although he was wrong when he claimed that zero divided by zero gives zero, however it was an excellent effort from his side which opened the gate of extension of Arithmetic. It was he who gave both the sum of the squares and cubes of first n natural numbers although no proof of the same was provided from him.

5. Bhaskaracharya (1114 AD – 1185 AD)
He is also known as Bhaskar II. He is said to have represent the peak of mathematical knowledge in the 12th century. His three important works are *Lilavati, Bijaganita* and *Siddhantasiromani*. In *Lilawati* (The beautiful), he explained eight mathematical operators which includes addition, subtraction, division, multiplication, squaring, cubing, extraction of square root of a number and extraction of cube root of a number. In *Bijaganita* (Seed Counting), he explained concepts of algebra. His work *Siddhantasiromani*is in two parts. The first part is a treatise on mathematical astronomy and the second part deals with sphere. He also tried to improve upon the attempts of Brahmagupta of division by zero but did not get much success. There is a story behind writing of Lilavati by him. It goes as follows –

"Lilavati was the name of Bhaskaracharya's daughter. From casting her horoscope, he discovered that the auspicious time for her wedding would be a particular hour on a certain day. He placed a cup with a small hole at the bottom of the vessel filled with water, arranged so that the cup would sink at the beginning of the propitious hour. When everything was ready and the cup was placed in the vessel, Lilavati suddenly out of curiosity bent over the vessel and a pearl from her dress fell into the cup and blocked the hole in it. The lucky hour passed without the cup sinking. Bhaskaracharya believed that the way to console his dejected daughter, who now would never get married, was to write her a manual of mathematics"

6. Conclusion
The middle period (400 AD – 1200 AD) is rightly called Golden period of Mathematics in India. In this period, there has been tremendous progress in the field of Mathematics and Astronomy. Several concepts like planetary motions, decimal systems, arithmetic progression, trigonometric formulas, operation on numbers, zero were invented in this period only. *Ganit, Bijaganita, KhagolShashtra*among others were developed phenomenally by Mathematicians of this era. After this period, the pace of the progress was stalled in India. This is a matter of further research as to find why there was slow progress in the field of Mathematics, Astronomy and Science in India. One can deliberate on question like why European nations took the lead after that and we lagged behind them.

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